A simple model of the hypersonic boundary layer

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Abstract

A simple model of the hypersonic boundary layer is proposed. There are three assumptions: The mean velocity profile is linear, the total enthalpy is uniform, and turbulent transport is controlled by sonic eddies, whose rotational Mach number is unity. The model predicts that turbulent transport is slowest at the outer edge of the layer, consistent with the formal assumption of a linear velocity profile. The concentration profile of any conserved scalar is uniform across the boundary layer, matching the boundary condition at the wall. Any difference with the free stream concentration is accommodated by a jump at the outer edge of the layer.

1. Introduction

A central question in the compressible boundary layer is the physical mechanism of turbulent transport of mass, momentum, and energy. The conventional view is that Mach number influences the boundary layer through the energy equation and hence the density (Van Driest 1951, Morkovin 1961). However, density is known to have only a weak effect on the free shear layer (Brown & Roshko 1974). It is difficult to see why this would not also be true for the boundary layer. A sonic eddy model assumes that acoustic signaling rather than density controls the physics (Breidenthal 1992). A nonsteady entrainment event can only occur if the rotation period of a vortex is equal to or greater than the acoustic signaling time across the vortex diameter. The largest such vortex is termed a sonic eddy, with a rotational Mach number of unity. According to the model, it dominates the turbulent transport. Its size λ* is approximately equal to the local speed of sound a divided by the local velocity gradient.

This model was recently extended to the boundary layer (Dintilhac & Breidenthal 2022). It is natural to explore the implications of that model in the hypersonic limit. To facilitate that end, three simplifying assumptions are made below.

2. Assumptions for an idealized hypersonic boundary layer

Linear mean velocity profile

Neeb *et al*. (2018) found that the mean velocity profile in a Mach 6 boundary layer is remarkably linear. The idealized mean velocity profile, $u/U_\infty = y/\delta$, is shown in Figure 1. The local speed is u, the free stream speed is U_∞, the distance from the wall is y, and the boundary layer thickness is δ.

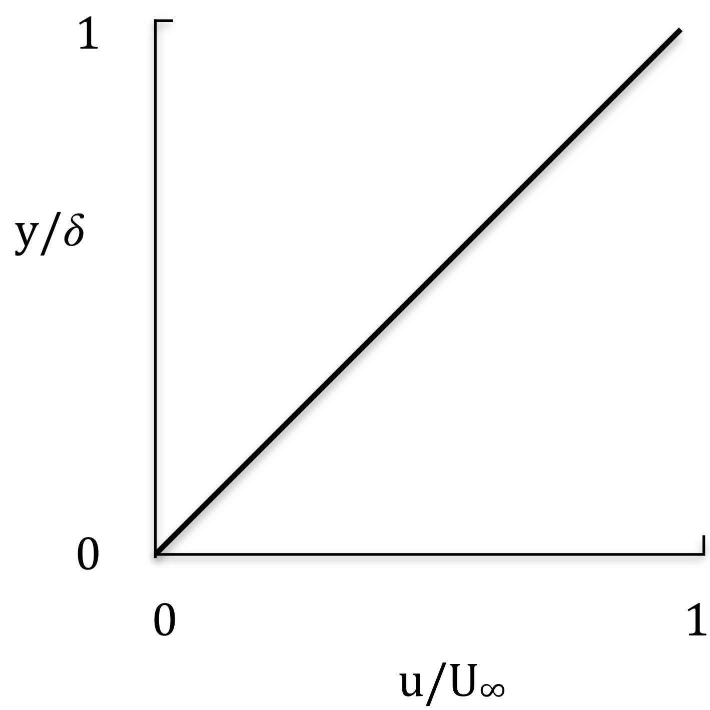

Figure 1. Assumed velocity profile

Uniform total enthalpy

In reality, the wall recovery temperature is somewhat less than the freestream stagnation temperature. So neither the total temperature, the total enthalpy, nor the total

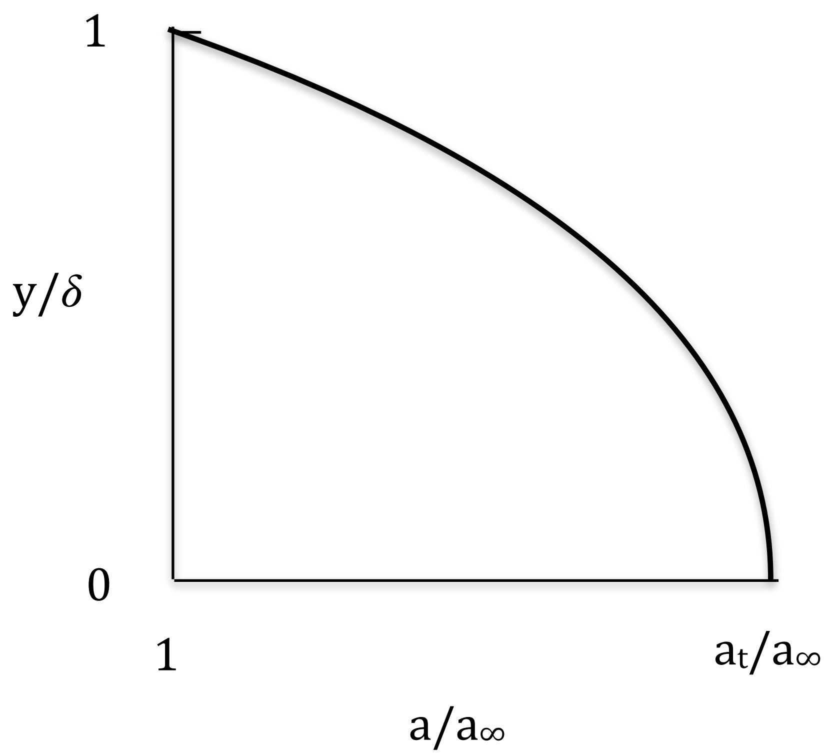

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Figure 2. *S*peed of sound profile

speed of sound a_t are uniform in reality. Here, however, they are all assumed uniform for simplicity, so $a_t = a_{t\infty}$ everywhere. The freestream speed of sound is a_∞, and the freestream stagnation speed of sound is $a_{t\infty} = a_t$.

Sonic eddy transport

According to the sonic eddy model, the largest active eddies have a rotational speed equal to local speed of sound. The sonic eddy size λ^* is the local speed of sound divided by the local velocity gradient, the latter assumed a constant, U/δ.

$$\lambda^*/\delta = a/U_\infty = [(a_t/U_\infty)^2 - (\gamma\text{-}1)\ (u/U_\infty)^2/2]^{1/2}, \qquad (1)$$

where

$$(a_t/U_\infty)^2 = (\gamma\text{-}1)/2 + 1/M_\infty^2. \qquad (2)$$

Since

$$u/U_\infty = y/\delta, \qquad (3)$$

$$\lambda^*/\delta = a/U_\infty = [(a_t/U_\infty)^2 - (\gamma\text{-}1)\ (y/\delta)^2/2]^{1/2}. \qquad (4)$$

The profile of the speed of sound is sketched in Figure 2, and the size of the sonic eddy is plotted in Figure 3.

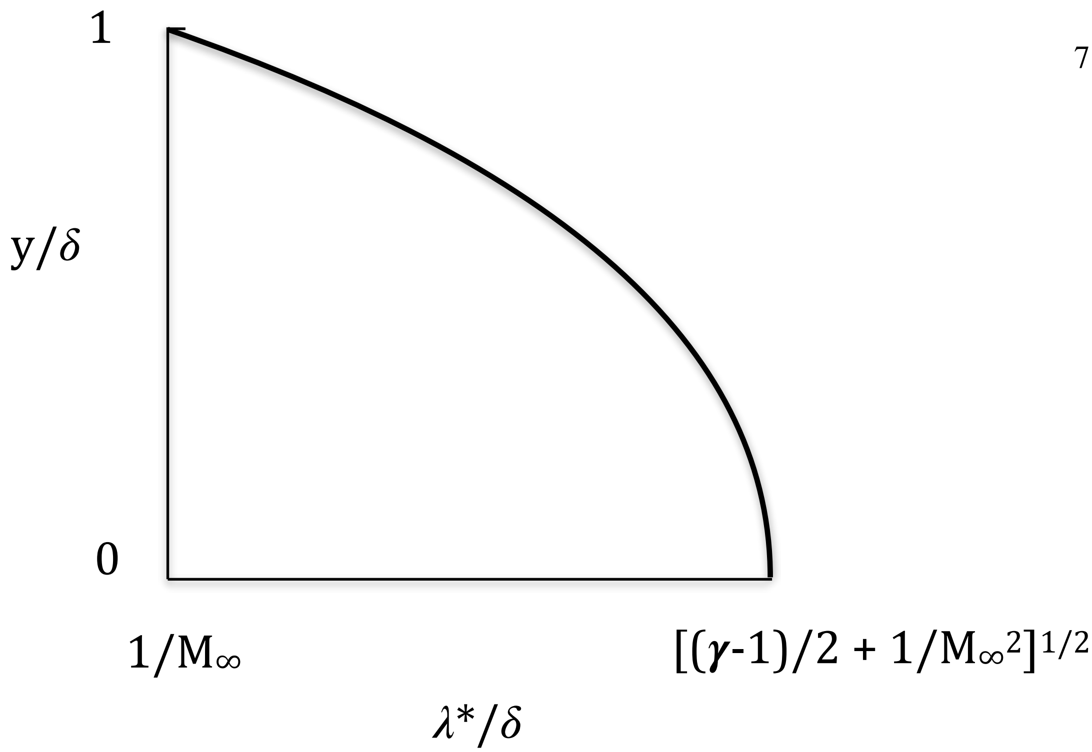

Figure 3. Profile of the sonic eddy size

Discussion

According to Figure 3, the size of the sonic eddy are both sharp minimum at the outer edge of the layer. Consequently, the momentum transport is slowest there. According to Corrsin (1974), gradient diffusion and eddy viscosity are only valid physical models if the largest eddies responsible for transport are all small compared to the distance in question. From Figure 3, it is clear that his criterion is easily satisfied in the outer part of the hypersonic boundary layer. Near the wall, however, his criterion can not be satisfied. The sonic eddies there are not small compared to the distance in question, the height above the wall. The boundary between these two domains is estimated by equating the size of the sonic eddy with the distance from the wall,

$$\lambda^*/\delta = y/\delta = y_d/\delta, \tag{5}$$

where y_d is the height above which turbulent diffusion is a valid model. From Equations (1) and (2),

$$y_d/\delta = [2/(\gamma + 1)]^{1/2}\, a_t/U_\infty = \{[(\gamma-1)/(\gamma+1)] + 2/[(\gamma+1)M_\infty^2]\}^{1/2}. \tag{6}$$

In the diffusion region, $y/\delta > y_d/\delta$, and the eddy diffusivity is $\varepsilon = a\lambda^*$. From Figures 1 and 2, it is obvious that both a and λ^* have a sharp minimum at the outer edge of the layer, $y/\delta = 1$. Consequently, this location must be a restrictive bottleneck for the rate of momentum transport from the free stream into the layer. Since the transport rate is much higher everywhere else inside the layer, the velocity there must approach a linear profile in the limit of large M_∞.

Momentum is transported at the outer edge of the boundary layer by the sonic eddies there, which have a characteristic velocity of a_∞. Therefore the skin friction coefficient must go as $1/M_\infty$, as noted by Dintilhac & Breidenthal (2022). Ironically, the turbulence at the outer edge of the boundary layer controls the skin friction at the wall.

For $y/\delta < y_d/\delta$, diffusion is not an appropriate physical model of turbulent transport. While the transport is still controlled by the sonic eddies, they are no longer small compared to the distance in question, the height above the surface.

Mass transport

For an inert, conserved scalar, the transport bottleneck at the outer edge of the boundary layer implies that the concentration of the conserved scalar abruptly jumps there to a uniform value within the layer determined by the boundary condition at the wall and the entrainment velocity at the outer edge. For example, in the limit of rapid consumption of the species at the wall, the uniform concentration would abruptly decline from the freestream value to zero within the layer. Conversely, the rapid generation of an inert scalar at the wall would generate a relatively high uniform concentration within the layer, abruptly falling to the freestream value at the outer edge. In the steady state, the jump would correspond to the rate of generation divided by the entrainment velocity of the sonic eddies at the outer edge. Since the latter is just the freestream speed of sound, the concentration jump is just the wall surface flux divided by a_∞.

Conclusions

An idealized hypersonic boundary layer suggests that turbulent transport is a sharp minimum at the outer edge. This bottleneck limits the transport of momentum into the layer, consistent with the formal assumption that the velocity profile is linear. The bottleneck implies that the concentration field of an inert scalar would be uniform within the layer, with a jump to the freestream value at the outer edge.

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